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PRODUCTION OF A NARROW BAND OF 0.511-MeV RADIATION BY USE OF THE PHERMEX BREMSSTRAHLUNG SPECTRUM

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ABSTRACT

The pair production cross section is numerically integrated over a typical PHERMEX bremsstrahlung spectrum to obtain the probability of pair production in a target of nuclear charge Z, and density ρ. The pair production cross section used herein is only approximate in that it (1) neglects screening, (2) neglects the Coulomb field for the emerging pair (first Born approximation), and (3) neglects pair production by atomic electrons. In spite of these approximations, the present work still gives an order-of-magnitude estimate of the amount of 0.511-MeV radiation produced by a typical pulse.

I. INTRODUCTION

Herein, the pair production cross section is numerically integrated over the PHERMEX output spectrum for a typical case given by Venable et al.[1] The pair production cross section for unpolarized photons of energy $\hbar\omega$ is,[2,3,4,5]

$$\sigma(\omega) = \alpha Z^2 r_o^2 \left\{ 2\eta^2[2C_2(\eta)-D_2(\eta)] + \frac{2}{27}[(109+64\eta^2)E_2(\eta)-(67+6\eta^2)(1-\eta^2)F_2(\eta)]\right\}, \quad (1)$$

where

$$C_2(\eta) = \int_1^{1/\eta} \frac{\cosh^{-1}x}{x}\cosh^{-1}\frac{1}{\eta x}\,dx \qquad (\eta \le 1), \quad (2)$$

$$D_2(\eta) = \int_1^{1/\eta} \frac{\cosh^{-1}\frac{1}{\eta x}}{\sqrt{x^2-1}}\,dx \qquad (\eta \le 1), \quad (3)$$

$$E_2(\eta) = F(\sqrt{1-\eta^2}) - E(\sqrt{1-\eta^2}) \qquad (\eta \le 1), \quad (4)$$

$$F_2(\eta) = F(\sqrt{1-\eta^2}) \qquad (\eta \le 1), \quad (5)$$

and $\eta = 2mc^2/\hbar\omega$. In Eq. (1) α is the fine structure constant, r_o is the classical radius of the electron, and Z is the nuclear charge. F and E in Eqs. (4) and (5) denote the complete elliptic integrals of the first and second kind, respectively;

$$F(\sqrt{1-\eta^2}) = \int_0^{\pi/2} \frac{d\phi}{\sqrt{1-(1-\eta^2)\sin^2\phi}}, \text{ and} \quad (6)$$

$$E(\sqrt{1-\eta^2}) = \int_0^{\pi/2} \sqrt{1-(1-\eta^2)\sin^2\phi}\,d\phi . \quad (7)$$

The basic integral over the PHERMEX bremsstrahlung output spectrum $P(\hbar\omega)$, is,

$$F = \int_{2mc^2}^{\hbar\omega max} P(\hbar\omega)\sigma(\hbar\omega)d(\hbar\omega), \quad (8)$$

where ωmax is the maximum frequency contained in $P(\hbar\omega)$ and m is the electron rest mass.

II. INTEGRATION TECHNIQUES

The actual integrations involved in Eqs. (2) - (8) are completed by Gauss-Legendre integration algorithms. All integrals are written in the form,

$$I = \int_{-1}^{1} f(y)dy = \sum_{i=1}^{m} a^j(i)\{f[y^j(i)]+f[-y^j(i)]\}, \quad (9)$$

where $a^j(i)$ and $y^j(i)$ are the Gauss-Legendre weights and coordinates, respectively.[6]

The weights $a^j(i)$ and the coordinates $y^j(i)$ are chosen so that Eq. (9) is exact when f(y) is a polynomial of degree 2m in y. All of the integrals involved in this calculation are accurately calculated by Gauss-Legendre sums with small values of m; i.e., all integrands are closely approximated by polynomials of low order.

III. POWER SPECTRUM

A typical power spectrum for the PHERMEX bremsstrahlung output[1] has been used in this work to estimate the value of Eq. (8). This normalized spectrum is defined by linear interpolation between the ordered pairs of energy and power spectrum intensity in Table I.

IV. PAIR PRODUCTION CODE

A FORTRAN code was written to evaluate Eq. (8) and the integrals in Eqs. (2) - (7). This program is capable of numerical integration of any function which is adequately approximated by a polynomial of degree 32 or less. In addition, any set of ordered pairs, as in Table I, is allowed in this code. The program, which is documented with comment cards, is listed in Table II and follows the notation of Eqs. (1) - (9). The final value calculated by the code must be multiplied by the factor $-\alpha Z^2 r_o^2$, which is contained in Eq. (1).

TABLE I

ENERGY SPECTRUM[a]

E_i(MeV)	P_i
0.0	.220
6.0	.175
12.0	.145
24.0	.105
27.0	.075
28.5	.050
29.4	.000

[a] (E_i, P_i) pairs represent the power spectrum used in evaluating Eq. (8). A typical power spectrum[1] was chosen for the present work.

V. PAIR PRODUCTION PROBABILITY

The probability that the normalized power spectrum will produce an electron-positron pair in the first millimeter of interaction with a target of atomic weight A, atomic number Z, and density ρ, is

$$P = F N_A \rho/A, \quad (10)$$

where

$$F = -\alpha Z^2 r_o^2 \text{ (Computer Output)}, \quad (11)$$

and N_A is Avogadro's number. Equation (10), of course, does not include the effects of Compton scattering on the photons represented by $P(\omega)$. The magnitude of the Compton scattering cross section is, for many elements, comparable with the pair production cross section.[5] However, it must be recalled that Compton scattering only redistributes the photon distribution and, thus, the only major influence on Eq. (1) is that some photons are scattered below the pair production threshold of $2mc^2$.

The Compton scattering contribution is relatively small for lead in a typical PHERMEX energy range and the pair production cross section dominates. Evaluating Eq. (10) for Pb, we find,

$$P = -\left(\frac{1}{137.}\right)(82)^2\ (2.8\text{x}10^{-13}\text{cm})^2\ (-14.48)$$

$$\text{x}\left(\frac{6.0225\text{x}10^{23}}{207}\ \frac{\text{atoms}}{\text{gram}}\right)\ (11.35\ \text{g/cm}^3) \quad (12)$$

$$\simeq 0.18/\text{mm}.$$

Thus the probability that the unity-normalized PHERMEX bremsstrahlung spectrum will produce a positron-electron pair is 0.18 for each mm of length of a Pb target. The normalization factor for the power spectrum (which should multiply Eq. (12) to give the number of positrons produced per mm) is given by the average number of photons per MeV in the PHERMEX output.

VI. PRODUCTION OF A NARROW BAND OF RADIATION FROM POSITRONS

Positrons produced by the above mechanism are stopped very quickly in Pb. This follows from (1) the Bethe stopping-power formula[5] which indicates that an electron of 50 MeV has an average range of 12.5 mm in Pb, (2) the fact that maximum energy of the positrons considered herein is less than the maximum bremsstrahlung energy (29.4 MeV was taken as the maximum in Table I), and (3) the observation that high-energy positrons behave electromagnetically as high-energy electrons.

The cross section for the annihilation of an electron-positron pair into two photons is,[7,5]

$$\sigma_{2\gamma} = \pi r_o^2 \frac{1}{\gamma+1}\left[\frac{\gamma^2+4\gamma+1}{\gamma^2-1}\ln\left(\gamma+\sqrt{\gamma^2-1}\right) - \frac{\gamma+3}{\sqrt{\gamma^2-1}}\right] \quad (13)$$

where $\gamma = \frac{1}{\sqrt{1-\beta^2}}$, $\beta = v/c$, and v is the velocity of the positron with respect to the electron at rest. Using the relations

$$\sqrt{\gamma^2-1} = \gamma\beta \quad (14a)$$

and

$$\ln\left(\gamma + \sqrt{\gamma^2-1}\right) = \frac{1}{2}\ln\left(\frac{1+\beta}{1-\beta}\right), \quad (14b)$$

and expanding the logarithm in Eq. (14b) we have,

$$\sigma_{2\gamma} = \pi r_o^2 \frac{1}{\gamma+1}\left[\frac{\gamma+1}{\gamma^2\beta^2}\beta + \frac{\gamma^2+4\gamma+1}{\gamma^2\beta^2}\left(\frac{1}{3}\beta^3+\frac{1}{5}\beta^5+\cdots\right)\right]$$

$$= \frac{\pi r_o^2}{\gamma^2\beta} + \pi r_o^2\frac{\gamma^2+4\gamma+1}{\gamma^2(\gamma+1)}\left(\frac{\tanh^{-1}\beta-\beta}{\beta^2}\right) \quad (15)$$

$$= \frac{\pi r_o^2}{\gamma^2\beta} + \pi r_o^2\frac{\gamma^2+4\gamma+1}{\gamma^2(\gamma+1)}\left(\frac{\beta}{3}+\frac{\beta^3}{5}\frac{\beta^5}{7}+\cdots\right) .$$

Upon expanding γ in Eq. (15) in terms of β we find,

$$\sigma_{2\gamma} = \frac{\pi r_o^2}{\beta} - \pi r_o^2\beta + \pi r_o^2\beta + \pi r_o^2\beta + \pi r_o^2\left(\frac{3}{5}-\frac{3}{4}\right)\beta^3 + 0(\beta^5)$$

$$= \frac{\pi r_o^2}{\beta} - \pi r_o^2\frac{3}{20}\beta^3 + 0(\beta^5) ; \quad (16)$$

i.e., the linear terms in β cancel. Equation (16) indicates that the major contribution to the two-photon annihilation of an electron-positron pair occurs for small values of β. Thus the kinetic energies of the annihilating particles are small compared to their rest mass. This implies that (1) annihilation will result in two photons of approximately 0.511-MeV energy in opposite directions and (2) the distribution of this narrow band of 0.511-MeV radiation will be isotropic allowing for any observation angle that suits the experimenter. The one-photon annihilation is, of course, forbidden by conservation of angular momentum. The three-photon cross section is smaller than the two-photon cross section of Eq. (13) by a factor of (1/137). Ref. 8 contains a review of the theoretical annihilation characteristics of electron-positron pairs for all order processes which have been observed or are likely to be observed for some time.

VII. DISCUSSION

As shown above, most of the photons in $P(\omega)$ will produce a positron if the target is several millimeters thick. This means that a large fraction of the energy in $P(\omega)$ will appear as 0.511-MeV radiation in a narrow band.

This narrow band of high-intensity radiation has not been exploited for any useful purpose at the PHERMEX facility. Among the various uses of this radiation are (1) the measurement of opacities at 0.511 MeV and (2) the measurement of solid state properties from a study of the exact annihilation spectrum. This last use has received considerable attention[9] and is commonly used to determine the solid state properties of the material in which the positrons are produced. The unique feature here is that these measurements would be made in the presence of an intense bremsstrahlung spectrum.

It is clear that not all of the 0.511-MeV γ rays produced in the Pb target will escape without interacting with the Pb target itself. At 0.511 MeV, γ rays interact with Pb by both Compton scattering and by the photoelectric effect.[5] The attenuation of 0.511-MeV radiation in Pb is described by an exponentially decreasing intensity, I:

$$I = I_o e^{-\tau x} ,$$

where I_o is the intensity at the point of production, x is the distance through which the γ rays travel in Pb and $\tau = 0.17$ per mm.[5] Thus the intensity of 0.511-MeV γ rays is diminished by the factor e^{-1} in about 6 mm of Pb. In comparison, an electron with 5 MeV (50 MeV) of kinetic energy has a range[5] of 3.3 mm (12.5 mm).

These data indicate that the optimum 0.511-MeV pulse will be obtained when some dimension of the Pb target is restricted to about 5 mm in extent: this thickness of Pb will stop most of the positrons produced in the PHERMEX energy range and will allow about 50% of the 0.511-MeV radiation produced by positron annihilation to escape unattenuated. A particularly attractive target design consists of an array of cylinders of Pb, each about 5 mm in diameter and 50 to 100 mm long, with their axes parallel to the PHERMEX beam. This target would provide a long interaction distance for the PHERMEX photons and would allow most of the 0.511-MeV radiation produced at 90° to the incident beam to escape the Pb.

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TABLE II

PAIR PRODUCTION CODE[a]

(LASL Identification: LP-0640)

```
      PROGRAM PUBLIS(INP,FSFT5=INP,OUT,FSFT6=OUT)
      DIMENSION Y(11,16),A(11,16),IM(11)
      COMMON Y,A
      NORD=11
C        HERE WE READ THE PARAMETERS FOR THE GAUSS-LEGENDRE
C        INTEGRATION ROUTINE
      DO 1 MM=1,NORD
      READ(5,100) N
 100  FORMAT(I10)
      IND=N/2
      IM(MM)=IND
      READ(5,101) (Y(MM ,I),A(MM ,I),I=1,IND)
  101 FORMAT(8F10.8)
    1 CONTINUE
      M=4
      SUM=0.0
      IND=IM(M)
C        HERE WE INTEGRATE THE PAIR PROD. CROSS SECTION OVER THE
C        POWER SPECTRUM OF THE INITIAL PHOTON DISTRIBUTION
      DO 3 I=1,IND
      SUM=SUM+A(M,I)*(F(Y(M,I))  +  F(-Y(M,I)))
    3 CONTINUE
      NPOINT=IM(M)*2
      WRITE(6,102) NPOINT,SUM
  102 FORMAT(1X,I4,3X,E15.8)
      STOP
      END

      FUNCTION F(Y)
C        OEMAX IS THE MAXIMUM FREQUENCY OF THE PHOTON DISTRIBUTION
C        EM IS THE ELECTRON REST MASS IN MEV
C        A*Y+B IS THE FREQUENCY OF THE PHOTON DISTRIBUTION
C        P REPRESENTS THE POWER DISTRIBUTION OF INITIAL PHPTONS
C        SIGMA IS THE TOTAL CROSS SECTION FOR PAIR PRODUCTION
      OEMAX=29.4
      EM=.511002
      A=(OEMAX-2.*EM)/2.
      B=(OEMAX+2.*EM)/2.
      F=A*P(A*Y+B)*SIGMA(2.*EM/(A*Y+B))
      WRITE(6,44) F
  44  FORMAT(1X,*F=*,E15.8)
      RETURN
      END

      FUNCTION SIGMA(ETA)
C        ETA IS 2M/OMEGA AND IS ALWAYS SMALLER THAN 1 FOR PAIR PROD.
      SIGMA=-2.*(ETA**2)*(2.*C2(ETA)-D2(ETA))
     1  -(2./27.)*((109.+64.*(ETA)**2)*(FA(ETA)-FA(ETA))
     2    -(67.+6.*(ETA)**2)*(1.-ETA**2)*FA(ETA))
      OMEGA1=1.02/ETA
      WRITE(6,492) SIGMA,ETA,OMEGA1
 492  FORMAT(1X,*SIGMA=*,E15.8,3X,*ETA=*,E15.8,3X,*OMEGA=*,E15.8)
      RETURN
      END
```

[a]FORTRAN code for integrating the power spectrum over the pair production cross section.

```
      FUNCTION P(CERFQ)
      DIMENSION E(7),SI(7)
C        THE INITIAL PHOTON POWER SPECTRUM IS READ IN AS SEVEN
C        PAIRS OF NUMBERS AND THE BELOW ROUTINE DOES A LINEAR FIT
C        TO THESE SEVEN ORDERED PAIRS         THE ENERGIES E(1) THRU
C        E(7) ARE ORDERED WITH THE SEVEN INTENSITIES SI(1) THRU SI(7)
      E(1)=0.0
      E(2)=6.0
      E(3)=12.0
      E(4)=24.0
      E(5)=27.0
      E(6)=28.5
      E(7)=29.4
      SI(1)=.220
      SI(2)=.175
      SI(3)=.145
      SI(4)=.105
      SI(5)=.075
      SI(6)=.050
      SI(7)=.000
      IF (CERFQ.GT.E(1)) K=1
      IF (CERFQ.GT.E(2)) K=2
      IF (CERFQ.GT.E(3)) K=3
      IF (CERFQ.GT.E(4)) K=4
      IF (CERFQ.GT.E(5)) K=5
      IF (CERFQ.GT.E(6)) K=6
      P=SI(K)+((SI(K+1)-SI(K))/(E(K+1)-E(K)))*(CERFQ-E(K))
      RETURN
      END

      FUNCTION C2(ETA)
      DIMENSION Y(11,16),A(11,16)
      COMMON Y,A
      COMMON /ERT1/ AAA
      AAA=ETA
      SU1=0.0
      MM=16
      NN=11
      DO 2 J=1,MM
      SU1=SU1+A(NN,J)*(F1(Y(NN,J))   +   F1(-Y(NN,J)))
    2 CONTINUE
      C2=SU1
      WRITE(6,50) C2
   50 FORMAT(1X,*C2=*,E15.8)
      RETURN
      END

      FUNCTION F1(U)
      COMMON /ERT1/ AAA
      ETA=AAA
      F1=ACOSH(.5*(1./ETA-1.)*U+.5*(1.+1./ETA))
     1  *ACOSH(1./(.5*(1.-ETA)*U+.5*(ETA+1.)))
     2   *(1./(U+(1.+1./ETA)/(1./ETA-1.)))
      RETURN
      END

      FUNCTION D2(ETA)
      DIMENSION Y(11,16),A(11,16)
      COMMON Y,A
      COMMON /ERT2/ BBB
      BBB=ETA
      SU1=0.0
      MM=16
      NN=11
      DO 2 J=1,MM
      SU1=SU1+A(NN,J)*(F2(Y(NN,J))   +   F2(-Y(NN,J)))
    2 CONTINUE
      D2=SU1
      WRITE(6,59) D2
   59 FORMAT(1X,*D2=*,E15.8)
      RETURN
      END
```

```
      FUNCTION F2(U)
      COMMON /ERT2/ BBB
      ETA=BBB
      F2=((1.-ETA)/(2.*ETA))*
     1  SQRT(1./(((.5*(1./ETA-1.)*U+.5*(1.+1./ETA))**2-1.))
     2   *ACOSH(1./(.5*(1.-ETA)*U+.5*(ETA+1.)))
      RETURN
      END

      FUNCTION FA(ETA)
      DIMENSION Y(11,16),A(11,16)
      COMMON Y,A
      COMMON /ERT3/ CCC
      CCC=ETA
      SU1=0.0
      MM=16
      NN=11
      DO 2 J=1,MM
      SU1=SU1+A(NN,J)*(F3(Y(NN,J))  +  F3(-Y(NN,J)))
    2 CONTINUE
      FA=SU1
      WRITE(6,62) FA
   62 FORMAT(1X,*F=*,E15.8)
      RETURN
      END

      FUNCTION F3(U)
      COMMON /ERT3/ CCC
      ETA=CCC
      PIOF=3.1415926538/4.
      ACBGTR=     (1.-ETA**2)
      F3=PIOF*  SQRT(1./(1.-(ACBGTR    )*SIN(PIOF*(1.+U))
     1  *SIN(PIOF*(1.+U))))
      RETURN
      END

      FUNCTION FA(ETA)
      DIMENSION Y(11,16),A(11,16)
      COMMON Y,A
      COMMON /ERT4/ DDD
      DDD=ETA
      SU1=0.0
      MM=16
      NN=11
      DO 2 J=1,MM
      SU1=SU1+A(NN,J)*(F4(Y(NN,J))  +  F4(-Y(NN,J)))
    2 CONTINUE
      FA=SU1
      WRITE(6,78) FA
   78 FORMAT(1X,*F=*,E15.8)
      RETURN
      END

      FUNCTION F4(U)
      COMMON /ERT4/ DDD
      ETA=DDD
      PIOF=3.1415926538/4.
      ACBGTR=     (1.-ETA**2)
      F4=PIOF*SQRT(1.-(ACBGTR    )*SIN(PIOF*(1.+U))
     1  *SIN(PIOF*(1.+U)))
      RETURN
      END

       FUNCTION ACOSH(XX)
       ACOSH=ALOG(XX+SQRT(XX**2-1.))
       RETURN
       END
$EM.
```

```
        2
.57735027 1.00000000
        4
.86113631 .34785485 .33998104 .65214515
        6
.93246951 .17132449 .66120939 .36076157 .23861919 .46791393
        8
.96028986 .10122854 .79666648 .22238103 .52553241 .31370665 .18343464 .36268378
       10
.97390653 .06667134 .86506337 .14945135 .67940957 .21908636 .43339539 .26926672
.14887434 .29552422
       12
.98156063 .04717534 .90411726 .10693933 .76990267 .16007833 .58731795 .20316743
.36783150 .23349254 .12523341 .24914705
       14
.98628381 .03511946 .92843488 .08015809 .82720131 .12151857 .68729290 .15720317
.51524864 .18553840 .31911237 .20519846 .10805495 .21526385
       16
.98940093 .02715246 .94457502 .06225352 .86563120 .09515851 .75540441 .12462897
.61787624 .14959599 .45801678 .16915652 .28160355 .18260341 .09501251 .18945061
       20
.07652652 .15275339 .22778585 .14917299 .37370609 .14209611 .51086700 .13168864
.63605368 .11819453 .74633191 .10193012 .83911697 .08327674 .91223443 .06267205
.96397193 .04060143 .99312860 .01761401
       24
.06405689 .12793820 .19111887 .12583746 .31504268 .12167047 .43379351 .11550567
.54542147 .10744427 .64809365 .09761865 .74012419 .08619016 .82000199 .07334648
.88641553 .05929858 .93827455 .04427744 .97472856 .02853139 .99518722 .01234123
       32
.04830767 .09654009 .14447196 .09563872 .23928736 .09384440 .33186860 .09117388
.42135128 .08765209 .50689991 .08331192 .58771576 .07819390 .66304427 .07234579
.73218212 .06582222 .79448380 .05868409 .84936761 .05099806 .89632116 .04283590
.93490608 .03427386 .96476226 .02539207 .98561151 .01627439 .99726386 .00701861
$FJ.
```